

## 3-1 Videos Guide

### 3-1a

Exercise:

Calculate, to four decimal places, the first ten terms of the sequence and use them to plot the graph of the sequence by hand. Does the sequence appear to have a limit? If so, calculate it. If not, explain why.

- $a_n = 2 + \frac{(-1)^n}{n}$

Theorem (statement):

- If  $\lim_{n \rightarrow \infty} |a_n| = 0$ , then  $\lim_{n \rightarrow \infty} a_n = 0$ .

Exercise:

- $a_n = 1 + \frac{10^n}{9^n}$

### 3-1b

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

- $a_n = \frac{3+5n^2}{1+n}$
- $a_n = \frac{4^n}{1+9^n}$
- $a_n = e^{2n/(n+2)}$

Theorems (statement):

- If  $\lim_{x \rightarrow \infty} f(x) = L$  and  $f(n) = a_n$  when  $n$  is an integer, then  $\lim_{x \rightarrow \infty} a_n = L$ .
- If  $\lim_{x \rightarrow \infty} a_n = L$  and the function  $f$  is continuous at  $L$ , then  $\lim_{x \rightarrow \infty} f(a_n) = f(L)$ .

### 3-1c

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

- $\left\{ \begin{array}{l} \ln n \\ \ln 2n \end{array} \right\}$
- $\frac{a_n(-3)^n}{n!}$

### 3-1d

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

- $a_n = \frac{1-n}{2+n}$

Theorem (statement):

- Monotonic Sequence Theorem: Every bounded, monotonic sequence is convergent.

3-1e

Exercise:

- Show that the sequence defined by

$$a_1 = 2, \quad a_{n+1} = \frac{1}{3-a_n}$$

satisfies  $0 \leq a_n \leq 2$  and is decreasing. Deduce that the sequence is convergent and find its limit.